Published by Institute of Physics Publishing for SISSA/ISAS
Received: *May 24, 2002*
Revised: *July 8, 2002*
Accepted: *July 11, 2002*

Non-maximally symmetric D-branes on group manifold in the lagrangian approach

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Abstract: Recently, Maldacena, Moore and Seiberg introduced non-maximally symmetric boundary states on group manifold using T-duality. In the work presented here we suggest simple description of these branes in terms of group elements. We show that T-dualization actually reduces to multiplication of conjugacy classes by the corresponding U(1) subgroups. Using this description we find the two-form trivializing the WZW three-form on the branes. SU(2) and $SL(2,\mathbb{R})$ examples are considered in details.

Keywords: Conformal Field Models in String Theory, D-branes.

JHEP07(2002)033

Contents

1. Introduction

During the last years, D-branes were studied using boundary conformal field theory. One of the most important criteria used in these studies was the number of symmetries preserved by the D-branes in question. Mainly were studied branes preserving the full chiral symmetry, which is usually broader than conformal symmetry.

Recently, some attempts were made to study branes preserving smaller symmetries. In [1]–[4] were constructed boundary states preserving only conformal symmetry.

In [5, 6], was developed a general approach for constructing boundary states of WZW models preserving only some part of the full affine symmetry by use of the T-duality in the directions of the Cartan subalgebra. Further, for the non-abelian subgroups, this approach was developed in [7]. But whenever a CFT target space receives a geometrical interpretation, the algebraically constructed brane can be realized as a geometrical subset. In [8], maximally symmetric branes in the WZW model are given by finite number of conjugacy classes. In [5] the shape of the non-maximally symmetric branes for the case of SU(2) group manifold was found, but this description left obscured the connection to underlying symmetries as well as, the possibility of generalization to other groups. An attempt to find similar branes on the $\mathrm{SL}(2,\mathbb{R})$ was done in [9].

In this work we suggest simple description of the non-maximally symmetric D-branes, derived by means of T-duality, in terms of group elements using symmetry arguments. We show that they are given by product of conjugacy classes with T-dualized U(1) subgroups.

We show consistent with work performed in [10, 8] and [11], that, in these branes, the WZW three-form belongs to the trivial cohomology class. We construct action with this boundary condition and show that it displays required symmetry.

We show that this action also can be derived by the direct T-dualization of the boundary WZW action.

Finally we show that for $G = \mathrm{SU}(2)$ location and mass of these branes coincide with corresponding values found in [5]. We also analyzed in detail, the location of these branes for the case of $\mathrm{SL}(2, \mathbb{R})$.

2. Algebraic description of T-dualized branes

In this section we briefly review construction of the non-maximally symmetric boundary state, introduced in [5] and [6].

The main idea of the construction is to represent group manifold G as orbifold $G = (G/H \times H)/\Gamma$, where H is an abelian subgroup of G, and then constructing boundary state for G as Γ-invariant linear combination of the tensors product of boundary states for coset G/H and for abelian subgroup H.

Let us illustrate this abstract construction for the case, when $G = \mathrm{SU}(2)_k$, $H = \mathrm{U}(1)_k$, $G/H = PF_k$ and $\Gamma = Z_k$:

$$\mathrm{SU}(2)_k = (PF_k \times \mathrm{U}(1)_k)/Z_k \,. \tag{2.1}$$

Before describing the results in depth, it may prove useful to review briefly the boundary states for $\mathrm{U}(1)_k$. Here as usual for the scalar field, we have the Neumann and Dirichlet boundary states. However extended symmetry present at the special values of the radius imposes some restrictions on the position of $D0$-brane for the Dirichlet boundary condition and on the Wilson line parameter of the $D1$-brane for the Neumann boundary condition. Using accepted notation A-branes and B-branes for the Dirichlet and Neumann boundary conditions correspondingly, the boundary states found in [5] are:

$$|A, n\rangle = \frac{1}{(2k)^{1/4}} \sum_{n'=0}^{2k-1} e^{\frac{-i\pi n n'}{k}} |A, n', n'\rangle\rangle \,, \tag{2.2}$$

for A-branes, and

$$|B, \eta = \pm 1\rangle = \left(\frac{k}{2}\right)^{1/4} [|B, 0, 0\rangle\rangle + \eta |B, k, -k\rangle\rangle] \,, \tag{2.3}$$

for B-branes, where $|A, n', n'\rangle\rangle$ and $|B, r, -r\rangle\rangle$ are A- and B-Ishibashi states correspondingly:

$$|Ar, r\rangle\rangle = \exp\left[+\sum_{n=1}^{\infty} \frac{\alpha_{-n}\tilde{\alpha}_{-n}}{n}\right] \sum_{l \in Z} \left| \frac{r + 2kl}{\sqrt{2k}}, \frac{r + 2kl}{\sqrt{2k}} \right\rangle , \tag{2.4}$$

$$|Br, -r\rangle\rangle = \exp\left[-\sum_{n=1}^{\infty} \frac{\alpha_{-n}\tilde{\alpha}_{-n}}{n}\right] \sum_{l \in Z} \left| \frac{r + 2kl}{\sqrt{2k}}, -\frac{r + 2kl}{\sqrt{2k}} \right\rangle . \tag{2.5}$$

We see that (2.2) describes $D0$-brane sitting at $2k$ special points, and (2.3) are two $D1$-branes with special values of the Wilson line parametrized by η.

Now we are in position to write down two kinds of boundary states for the $\mathrm{SU}(2)_k$ according to the main prescription:

$$|A, j\rangle = \frac{1}{\sqrt{k}} \sum_n |Ajn\rangle^{\mathrm{PF}} |A, n\rangle \tag{2.6}$$

and

$$|B, j, \eta = \pm 1\rangle = \frac{1}{\sqrt{k}} |B, \eta\rangle \sum_{n=0}^{2k-1} |Ajn\rangle^{\mathrm{PF}}, \tag{2.7}$$

where $|Ajn\rangle^{\mathrm{PF}}$ is the usual Cardy state for the parafermionic theory. It is easy to check that the Dirichlet gluing condition (2.6) just gives us the usual maximally symmetric Cardy state for the $\mathrm{SU}(2)_k$ affine algebra, but (2.7) gives us new non-maximally symmetric branes. From the form of the boundary state we see that preserved symmetry now is diagonal $\mathrm{U}(1)$ (and a Z_k). Using formula (2.7) it is easy to compute mass and shape of the new branes.

In [5] the mass was found by the overlap of the (2.7) with $|A, j = 0\rangle\rangle$:

$$M(Bj) = \sqrt{k} M(Aj)\,. \tag{2.8}$$

Then by the overlap of the (2.7) with the graviton wave packet was found the shape of the branes:

$$\langle B, j, \eta|\tilde{\theta}\rangle \sim k \sum_j D^{j'}_{00} S_{jj'} \sim i e^{i\hat{\psi}} \sum_{n=0}^{\infty} P_n(\cos\tilde{\theta}) e^{in2\hat{\psi}} + \text{c.c.} \sim \frac{\Theta(\cos\tilde{\theta} - \cos 2\hat{\psi})}{\sqrt{\cos\tilde{\theta} - \cos 2\hat{\psi}}}, \tag{2.9}$$

where $D^{j'}_{mm'}$ is matrix of rotations, P_n are Legendre polynomials, $\tilde{\theta}$ is the second Euler angle, $\hat{\psi} = 2j\pi/k$, $S_{jj'}$ is matrix of modular transformation for $\mathrm{SU}(2)_k$, and $\Theta(z)$ is the usual step function which vanishes when $z < 0$. We see that generically these are three-dimensional branes covering only part of the group manifold. But for even values of k, T-dualizing the biggest equatorial conjugacy class with $j = k/4$ results in brane covering the whole group manifold. In [5] it was conjectured that the partially covering branes are unstable, while the last one is stable. By T-dualizing D0-brane, a D1-brane is formed along a maximum circle of S^3, which is unstable.

It is straightforward to generalize construction of the (2.7) to that of other groups [6]. Let us consider for example unitary group $G = \mathrm{SU}(N+1)$ and the embedding $\mathrm{SU}(N) \times \mathrm{U}(1) \hookrightarrow \mathrm{SU}(N+1)$, where $\mathrm{U}(1)$ corresponds to the generator

$$H^N = \frac{1}{\sqrt{N(N+1)}} \mathrm{Diag}\{1, \ldots, 1, -N\}\,.$$

It was shown in [6] that performing T-duality with respect to the current H^N we get boundary state $|B\rangle$, satisfying the boundary conditions:

$$(J^a_n + \tilde{J}^a_{-n})|B\rangle = 0 \qquad a \in \mathrm{su}(N)\,, \tag{2.10}$$

$$(H^N_n - \tilde{H}^N_{-n})|B\rangle = 0\,. \tag{2.11}$$

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However this does not satisfy simple boundary conditions with respect to the remaining currents. Therefore preserved symmetry is now $\mathrm{SU}(N)_{\mathrm{vectorial}} \times \mathrm{U}(1)_{\mathrm{axial}}$, where subscripts vectorial and axial refer to the signs plus and minus in (2.10) and (2.11) respectively.

3. Geometrical description of the non-maximally symmetric branes

3.1 Definition

In this section we present the main result of this work, which is the geometrical description of the T-dualized boundary state reviewed in the previous section.

It is useful to begin with the Polyakov-Wiegmann identities which will be referred to frequently in this section.

$$\begin{aligned} L^{\mathrm{kin}}(gh) &= L^{\mathrm{kin}}(g) + L^{\mathrm{kin}}(h) - \left(\mathrm{Tr}\left(g^{-1}\partial_z g \partial_{\bar{z}} h h^{-1} + \mathrm{Tr}\left(g^{-1}\partial_{\bar{z}} g \partial_z h h^{-1}\right)\right)\right), \quad (3.1)\\ \omega^{\mathrm{WZ}}(gh) &= \omega^{\mathrm{WZ}}(g) + \omega^{\mathrm{WZ}}(h) - d\left(\mathrm{Tr}\left(g^{-1}dgdhh^{-1}\right)\right), \quad (3.2)\end{aligned}$$

where $L^{\mathrm{kin}} = \mathrm{Tr}(\partial_z g \partial_{\bar{z}} g^{-1})$ and $\omega^{\mathrm{WZ}} = \frac{1}{3}\mathrm{Tr}(g^{-1}dg)^3$.

We define the new D-branes as product of the conjugacy class with the U(1) subgroup. In other words we study the following boundary condition:

$$g_{|\mathrm{boundary}} = Lhfh^{-1}, \quad (3.3)$$

where $L \in \mathrm{U}(1)$, $f = e^{\frac{2\pi}{k}\lambda\cdot H}$, H are Cartan generators, λ is a vector in the weight lattice, $h \in G$. We denote $C = hfh^{-1}$.

Recalling that, according to the analysis of [10], and [11], in order for some subset to be a good boundary condition for the WZW model, restriction of the WZW three-form to that subset should belong to the trivial cohomology class. In other words, on the subset should exist a two-form $\omega^{(2)}$ satisfying the condition:

$$d\omega^{(2)} = \omega^{\mathrm{WZ}}. \quad (3.4)$$

It may be easily checked that proposed branes satisfy those criteria. At the boundary (3.3) according to (3.2)

$$\omega^{\mathrm{WZ}}(g) = \omega^{\mathrm{WZ}}(L) + \omega^{\mathrm{WZ}}(C) - d\,\mathrm{Tr}(L^{-1}dLdCC^{-1}). \quad (3.5)$$

Using that for the abelian group, L, $\omega^{\mathrm{WZ}}(L) = 0$, and

$$\omega^{\mathrm{WZ}}(C) = d\omega^f(h) = d\,\mathrm{Tr}(h^{-1}dhfh^{-1}dhf^{-1}), \quad (3.6)$$

we get

$$\omega^{\mathrm{WZ}}(g)_{|\mathrm{boundary}} = d\omega^{(2)}(L,h), \quad (3.7)$$

where

$$\omega^{(2)}(L,h) = \omega^f(h) - \mathrm{Tr}(L^{-1}dLdCC^{-1}). \quad (3.8)$$

3.2 Action and symmetry check

The action is defined as:

$$S = \frac{k_G}{4\pi}\left[\int_\Sigma d^2 z L^{\rm kin} + \int_B \omega^{\rm WZ} - \int_D \omega^{(2)}(L,h)\right], \tag{3.9}$$

where $\partial B = \Sigma + D$.

The boundary is invariant under the following list of transformations.

1. $g \to kgk^{-1}$ satisfying to the condition $[k, L] = 0$. Under this transformation $h \to kh$ and $C \to kCk^{-1}$. This means that for example, in the case of SU$(N+1)$, $k \in$ SU$(N) \times$ U(1).
2. $g \to kg$, where $k \in$ U(1). Under this transformation $L \to kL$.
3. $g \to gk$, where $k \in$ U(1). Under this transformation $L \to Lk$, $C \to k^{-1}Ck$ and $h \to k^{-1}h$. It follows from (2) and (3) that the boundary is invariant also under their axially diagonal combination:
4. $g \to kgk$, where $k \in$ U(1). Under this transformation $L \to kLk$, $C \to k^{-1}Ck$ and $h \to k^{-1}h$.

Using the method developed in [12], now we will show, that the action (3.9) is invariant under the following symmetries:

1. $$g(z,\bar z) \to h_L(z) g(z,\bar z) h_R^{-1}(\bar z)\,, \tag{3.10}$$
   with $h_L(z)|_{\rm boundary} = h_R(\bar z)|_{\rm boundary} = k(\tau)$, $k \in$ SU(N), in agreement with (2.10).
2. $$g(z,\bar z) \to h_L(z) g(z,\bar z) h_R(\bar z)\,, \tag{3.11}$$
   with $h_L(z)|_{\rm boundary} = h_R(\bar z)|_{\rm boundary} = k(\tau)$, $k \in$ U(1), in agreement with (2.11). It is important to note that in (3.10) we used vectorial combination of the left and right symmetries, whereas in (3.11) axial combination is used, in agreement with the sign difference between (2.10) and (2.11).

Under the transformation (3.10), the change in the $L^{\rm kin}$ term in (3.9) read from (3.1), is canceled by the corresponding Σ integral of the boundary term from the change in the $\omega^{\rm WZ}$ term, read from (3.2). In the presence of a world sheet boundary there remains the contribution from D to the latter change,

$$\Delta(S^{\rm kin} + S^{\rm WZ}) = \frac{k_G}{4\pi}\int_D {\rm Tr}[k^{-1}dk(gk^{-1}dkg^{-1} - g^{-1}dg - dgg^{-1})], \tag{3.12}$$

where $g = LC$. Substituting this value of g in (3.12) we get:

$$\Delta(S^{\rm kin} + S^{\rm WZ}) = \tag{3.13}$$
$$\frac{k_G}{4\pi}\int_D {\rm Tr}\left[k^{-1}dk(LCk^{-1}dkC^{-1}L^{-1} - C^{-1}L^{-1}(dLC + LdC) - (dLC + LdC)C^{-1}L^{-1})\right],$$

and using $[k, L] = 0$ and cyclic permutation under the trace we obtain:

$$\Delta(S^{\rm kin}+S^{\rm WZ}) = \frac{k_G}{4\pi}\int_D \mathrm{Tr}[k^{-1}dk(Ck^{-1}dkC^{-1}-C^{-1}L^{-1}dLC-C^{-1}dC-dLL^{-1}-dCC^{-1})]\,. \tag{3.14}$$

Now we compute $\omega^{(2)}(L, kh) - \omega^{(2)}(L, h)$, using that

$$\omega^f(kh) - \omega^f(h) = \mathrm{Tr}\left[k^{-1}dk(Ck^{-1}dkC^{-1} - C^{-1}dC - dCC^{-1})\right] \tag{3.15}$$

and

$$\mathrm{Tr}\left[L^{-1}dLd(kCk^{-1})kC^{-1}k^{-1} - L^{-1}dLdCC^{-1}\right] = \mathrm{Tr}\left[L^{-1}dLdkk^{-1} - L^{-1}dLCk^{-1}dkC^{-1}\right], \tag{3.16}$$

resulting in

$$\begin{aligned}&\omega^{(2)}(L, kh) - \omega^{(2)}(L, h) = \\ &\qquad \mathrm{Tr}\left[k^{-1}dk\left(Ck^{-1}dkC^{-1} - C^{-1}dC - dCC^{-1} + L^{-1}dL - C^{-1}L^{-1}dLC\right)\right].\end{aligned} \tag{3.17}$$

Collecting (3.14) and (3.17) we obtain:

$$\Delta S = \frac{k_G}{2\pi}\int_D \mathrm{Tr}(L^{-1}dLk^{-1}dk)\,. \tag{3.18}$$

Noting, that for $k \in \mathrm{SU}(N)$ and $L \in \mathrm{U}(1)$ $\mathrm{Tr}(L^{-1}dLk^{-1}dk) = 0$, we prove that the action (3.9) possesses by the vectorially diagonal $\mathrm{SU}(N)$ symmetry. We also see from (3.18) that the vectorially diagonal $\mathrm{U}(1)$ symmetry is broken.

Now we will show that the action (3.9) possesses by the axially diagonal $\mathrm{U}(1)$ symmetry (3.11). By the same arguments, leading to the (3.12), we get that in the presence of the boundary under (3.11):

$$\Delta(S^{\rm kin} + S^{\rm WZ}) = \frac{k_G}{4\pi}\int_D \mathrm{Tr}[k^{-1}dk(g^{-1}dg - gk^{-1}dkg^{-1} - dgg^{-1})]\,, \tag{3.19}$$

where $g = LC$. Substituting this value of g in (3.19) we get:

$$\Delta(S^{\rm kin}+S^{\rm WZ}) = \frac{k_G}{4\pi}\int_D \mathrm{Tr}[k^{-1}dk(C^{-1}dC-Ck^{-1}dkC^{-1}+C^{-1}L^{-1}dLC-dLL^{-1}-dCC^{-1})]\,. \tag{3.20}$$

Now we compute $\omega^{(2)}(kLk, k^{-1}h) - \omega^{(2)}(L, h)$, using that

$$\omega^f(k^{-1}h) - \omega^f(h) = \mathrm{Tr}[k^{-1}dk(Ck^{-1}dkC^{-1} + C^{-1}dC + dCC^{-1})] \tag{3.21}$$

and

$$\begin{aligned}&\mathrm{Tr}[(kLk)^{-1}d(kLk)d(k^{-1}Ck)k^{-1}C^{-1}k - L^{-1}dLdCC^{-1}] = \\ &\qquad \mathrm{Tr}[k^{-1}dk(2dCC^{-1} + 2Ck^{-1}dkC^{-1} + L^{-1}dL - C^{-1}L^{-1}dLC)]\,,\end{aligned} \tag{3.22}$$

resulting in

$$\begin{aligned}&\omega^{(2)}(L, kh) - \omega^{(2)}(L, h) = \\ &\qquad \mathrm{Tr}[k^{-1}dk(C^{-1}dC - Ck^{-1}dkC^{-1} - dCC^{-1} - L^{-1}dL + C^{-1}L^{-1}dLC)]\,,\end{aligned} \tag{3.23}$$

which cancels (3.20).

3.3 T-duality

In this subsection we give alternative derivation of the form $\omega^{(2)}(L,h)$ explaining its relation to the T-duality.

Remembering how to get the T-dual WZW action in the absence of boundary, as shown in [13], we parametrise group element as a product

$$g = L^{-1}p = e^{-i\phi H^N}p\,, \tag{3.24}$$

where H^N is a generator of the Lie algebra, then, using the Polyakov-Wiegmann identity, separate ϕ and p parts, and afterwards T-dualise the scalar part. In the presence of the boundary as we will see this procedure will be modified by boundary terms.

Considering boundary WZW action with conjugacy class as boundary condition, $g_{|\text{boundary}} = hfh^{-1}$:

$$S = \frac{k_G}{4\pi}\left[\int_\Sigma d^2zL^{\text{kin}} + \int_B \omega^{\text{WZ}} - \int_D \omega^f(h)\right]. \tag{3.25}$$

As was established in [8] and [11] with $f = e^{\frac{2\pi}{k}\lambda\cdot H}$, where H are Cartan generators, and λ is a vector in the weight lattice, (3.25) is a well defined action.

Inserting g in the form (3.24) to (3.25) after using the Polyakov-Wiegamnn identities (3.1), (3.2) we obtain:

$$\begin{aligned} S = \frac{k_G}{4\pi}\Bigg[&\int_\Sigma d^2zL^{\text{kin}}(p) + \int_B \omega^{\text{WZ}}(p) - \int_D (\omega^f(h) - \text{Tr}(L^{-1}dLdpp^{-1})) + \\ &+ \int_\Sigma \partial_z\phi\partial_{\bar z}\phi - 2i\int_\Sigma \partial_{\bar z}\phi\,\text{Tr}(H^N\partial_z pp^{-1})\Bigg]. \end{aligned} \tag{3.26}$$

At this point, boundary condition for ϕ should be specified. From (3.24) we see that if ϕ satisfies to the Dirichlet boundary condition, p at the boundary lies in the usual conjugacy class, but if ϕ satisfies to the Neumann boundary condition, p at the boundary takes value $p = Lhfh^{-1}$. In other words, it lies in the above discussed branes (3.3).

After short algebra it may be checked, that the integrand of the boundary integral in (3.26) equals to (3.8):

$$\begin{aligned} \text{Tr}(L^{-1}dLdpp^{-1}) &= \text{Tr}(L^{-1}dLd(LC)C^{-1}L^{-1}) = \text{Tr}(L^{-1}dL(dLC + LdC)C^{-1}L^{-1}) = \\ &= \text{Tr}(L^{-1}dLdLL^{-1}) + \text{Tr}(L^{-1}dLdCC^{-1}) = \text{Tr}(L^{-1}dLdCC^{-1})\,, \end{aligned} \tag{3.27}$$

where we used that $\text{Tr}(L^{-1}dLdLL^{-1}) = 0$ for the abelian group. This computation shows that (3.26) is actually the sum of the action (3.9),with the new branes as the boundary condition, with scalar field coupled to current. Since, as noted above, the action (3.25), for f chosen as above, is well-defined WZW action, it is proven that with the same choice of f also (3.9) is well defined WZW action.

4. Examples

4.1 Branes on SU(2)

Let us consider now the case g = SU(2) in details. It is convenient to parametrise the group element as

$$g = x_0\sigma_0 + i(x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3)\,, \tag{4.1}$$

with $x_0^2 + x_1^2 + x_2^2 + x_3^2 = 1$. In this parametrisation conjugacy class given as $x_0 = \cos\hat{\psi}$ where $\hat{\psi} = \frac{2\pi j}{k}$. This parametrisation connected with the Euler angles

$$g = e^{i\chi\frac{\sigma_3}{2}} e^{i\tilde{\theta}\frac{\sigma_1}{2}} e^{i\phi\frac{\sigma_3}{2}} \tag{4.2}$$

by formulae

$$\begin{aligned} x_0 &= \cos\frac{\tilde{\theta}}{2}\cos\frac{\chi+\phi}{2} \\ x_1 &= \sin\frac{\tilde{\theta}}{2}\cos\frac{\chi-\phi}{2} \\ x_2 &= \sin\frac{\tilde{\theta}}{2}\sin\frac{\chi-\phi}{2} \\ x_3 &= \cos\frac{\tilde{\theta}}{2}\sin\frac{\chi+\phi}{2}\,. \end{aligned} \tag{4.3}$$

We note that $x_0^2 + x_3^2 = \cos^2\frac{\tilde{\theta}}{2}$ and $x_1^2 + x_2^2 = \sin^2\frac{\tilde{\theta}}{2}$.

If one parametrises the U(1) subgroup as $e^{i\alpha\sigma_3}$ the D-branes are located at:

$$e^{i\alpha\sigma_3}(\cos\hat{\psi}\sigma_0 + i(x_1\sigma_1 + x_2\sigma_2 + x_3\sigma_3)) = \tilde{x}_0\sigma_0 + i(\tilde{x}_1\sigma_1 + \tilde{x}_2\sigma_2 + \tilde{x}_3\sigma_3), \tag{4.4}$$

where

$$\begin{aligned} \tilde{x}_0 &= \cos\hat{\psi}\cos\alpha - x_3\sin\alpha \\ \tilde{x}_1 &= x_1\cos\alpha + x_2\sin\alpha \\ \tilde{x}_2 &= x_2\cos\alpha - x_1\sin\alpha \\ \tilde{x}_3 &= x_3\cos\alpha + \cos\hat{\psi}\sin\alpha\,. \end{aligned} \tag{4.5}$$

We see that $\tilde{x}_1^2 + \tilde{x}_2^2 = x_1^2 + x_2^2$. From one side as we noted above $\tilde{x}_1^2 + \tilde{x}_2^2 = \sin^2\frac{\tilde{\theta}}{2}$, from the other side maximum value of the $x_1^2 + x_2^2$ on the conjugacy class is obviously $\sin^2\hat{\psi}$. So we have that on the new branes

$$\sin^2\frac{\tilde{\theta}}{2} \leq \sin^2\hat{\psi}\,. \tag{4.6}$$

Using that $2\sin^2\alpha = 1 - \cos 2\alpha$, we get that on the branes

$$\cos\tilde{\theta} \geq \cos 2\hat{\psi}\,, \tag{4.7}$$

which is exactly (2.9). It is useful to think about the new D-branes also as a collection of translated conjugacy classes along the whole U(1) subgroup. From this we get that their volume equals to the product of the radius of the U(1) subgroup and the volume of the conjugacy class. This perfectly matches to the mass formula (2.8). We also note that for $j = 0, k/2$ formula (3.3) gives us $D1$-brane along the U(1) subgroup, also in accordance with the algebraic analysis of section 2.

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4.2 New branes on $\mathrm{SL}(2,\mathbb{R})$

Now let us turn to the case of $\mathrm{SL}(2,\mathbb{R})$. A general group element can be parametrized as follows:

$$g = \begin{pmatrix} x_0 + x_1 & x_2 + x_3 \\ x_2 - x_3 & x_0 - x_1 \end{pmatrix}, \tag{4.8}$$

where

$$x_0^2 - x_1^2 - x_2^2 + x_3^2 = 1\,. \tag{4.9}$$

The conjugacy class is given by the condition $x_0 = C$.

Here, due to compactness of the subgroup generated by the σ_2 and non-compactness of the the subgroup generated by the σ_3 we have two inequivalent directions for which can be taken the Neumann boundary condition.

Considering first the non-compact case, if we parametrise the U(1) subgroup as $e^{\alpha\sigma_3}$ the shape of the branes will be given by:

$$e^{\alpha\sigma_3} g = \begin{pmatrix} e^{\alpha}(C + x_1) & e^{\alpha}(x_2 + x_3) \\ e^{-\alpha}(x_2 - x_3) & e^{-\alpha}(C - x_1) \end{pmatrix} = \begin{pmatrix} \tilde{x}_0 + \tilde{x}_1 & \tilde{x}_2 + \tilde{x}_3 \\ \tilde{x}_2 - \tilde{x}_3 & \tilde{x}_0 - \tilde{x}_1 \end{pmatrix}. \tag{4.10}$$

We see that

$$\tilde{x}_2^2 - \tilde{x}_3^2 = x_2^2 - x_3^2\,. \tag{4.11}$$

Rewriting eq. (4.9) in the form

$$x_3^2 - x_2^2 = (1 - C^2) + x_1^2\,, \tag{4.12}$$

and using (4.11) we can describe the branes by the following inequality:

$$\tilde{x}_3^2 - \tilde{x}_2^2 \geq 1 - C^2\,. \tag{4.13}$$

This inequality can be simplified using the Euler angle parametrisations described in the appendix. In the patch given by the parametrization (A.1) it can be written as

$$\sin^2\frac{\theta}{2} \geq 1 - C^2\,, \tag{4.14}$$

or

$$\cos\theta \leq 2C^2 - 1\,. \tag{4.15}$$

In the patch given by formulae (A.3) it can be written as

$$-\sinh^2\tau \geq 1 - C^2\,, \tag{4.16}$$

or

$$\cosh\tau \leq 2C^2 - 1\,. \tag{4.17}$$

Now let us turn to the case when we choose the Neumann boundary condition for the subgroup generated by the σ_2. Parametrising now the $\mathrm{U}(1) = e^{i\alpha\sigma_2}$, we find that the branes are located at:

$$e^{i\alpha\sigma_2} \begin{pmatrix} C + x_1 & x_2 + x_3 \\ x_2 - x_3 & C - x_1 \end{pmatrix} = \begin{pmatrix} \tilde{x}_0 + \tilde{x}_1 & \tilde{x}_2 + \tilde{x}_3 \\ \tilde{x}_2 - \tilde{x}_3 & \tilde{x}_0 - \tilde{x}_1 \end{pmatrix}, \tag{4.18}$$

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where

$$\begin{aligned}\tilde{x}_0 &= C\cos\alpha - x_3\sin\alpha \\ \tilde{x}_1 &= x_1\cos\alpha + x_2\sin\alpha \\ \tilde{x}_2 &= x_2\cos\alpha - x_1\sin\alpha \\ \tilde{x}_3 &= x_3\cos\alpha + C\sin\alpha. \end{aligned} \tag{4.19}$$

We see that

$$\tilde{x}_1^2 + \tilde{x}_2^2 = x_1^2 + x_2^2 \,. \tag{4.20}$$

Rewriting eq. (4.9) in the form

$$x_1^2 + x_2^2 = (C^2 - 1) + x_3^2 \,, \tag{4.21}$$

and using (4.20) we see that this brane can be described by the inequality:

$$\tilde{x}_1^2 + \tilde{x}_2^2 \geq C^2 - 1 \,. \tag{4.22}$$

Using now parametrisation (A.5) we get for the brane location:

$$\cosh\rho \geq 2C^2 - 1 \,. \tag{4.23}$$

5. Discussion

Here we outline some directions for the future work, which may further clarify properties of the T-dualized branes.

1. As was noted in [14] and [15] if action for the boundary WZW model is given in the form (3.25), it actually fixes also the two-form field strength on the D-brane world-volume by the formula:

   $$2\pi F = \omega^{(2)} - B \,, \tag{5.1}$$

   and the Born-Infeld action correspondingly has the form:

   $$S = \int \sqrt{\det(G + \omega^{(2)})} \,. \tag{5.2}$$

   If, for example to use corresponding formulae for the case of the maximally symmetric conjugacy class, we will get, as found in [16], brane-stabilizing magnetic monopole. Using the formula (3.8) we can compute the two-form field strength also for the new branes. This observation may be can help us firmly establish the stability properties of the branes mentioned in section 2.

2. In [17] it was noted that the conjugacy classes also arise as intersection of the $D4$ and $D6$-branes with SU(2) group manifold in the near-horizon limit of the $NS5$-brane. It is easy to check that corresponding intersection of the D-brane with the $NS5$-brane wrapping whole SU(2), or by other words containing all directions transverse to the $NS5$-brane, breaks all supersymmetries. In any case, it may be possible to find some intersection of that kind which is nevertheless stable. This also provides indirect evidence of the stability of the whole group covering branes.

3. It would be interesting to construct explicitly boundary state for the $SL(2,\mathbb{R})$ describing the new branes found in section 4.

Acknowledgments

Author thanks C. Bachas, S. Elitzur and A. Giveon for useful discussions. Author is especially grateful to S. Elitzur for a careful reading of the manuscript and discussion in section 4.

A. Euler angles for the $SL(2,\mathbb{R})$

Let us write for further applications connection of (4.8) to the Euler parametrisations [18]. For the case of the $SL(2,\mathbb{R})$ one has the different Euler parametrisations covering different patches of the group. One convenient Euler parametrisation of $g \in SL(2,\mathbb{R})$ is

$$g = e^{\chi \frac{\sigma_3}{2}} e^{i\theta \frac{\sigma_2}{2}} e^{\phi \frac{\sigma_3}{2}} \,. \tag{A.1}$$

It is connected to (4.8) by formulae

$$\begin{aligned}
x_0 &= \cos\frac{\theta}{2}\cosh\frac{\chi+\phi}{2} \\
x_1 &= \cos\frac{\theta}{2}\sinh\frac{\chi+\phi}{2} \\
x_2 &= \sin\frac{\theta}{2}\sinh\frac{\chi-\phi}{2} \\
x_3 &= \sin\frac{\theta}{2}\cosh\frac{\chi-\phi}{2} \,.
\end{aligned} \tag{A.2}$$

We see that $x_0^2 - x_1^2 = \cos^2\frac{\theta}{2}$ and $x_3^2 - x_2^2 = \sin^2\frac{\theta}{2}$. The second Euler parametrisation is

$$g = e^{\chi \frac{\sigma_3}{2}} e^{\tau \frac{\sigma_1}{2}} e^{\phi \frac{\sigma_3}{2}} \,. \tag{A.3}$$

It is connected to (4.8) by formulae

$$\begin{aligned}
x_0 &= \cosh\frac{\tau}{2}\cosh\frac{\chi+\phi}{2} \\
x_1 &= \cosh\frac{\tau}{2}\sinh\frac{\chi+\phi}{2} \\
x_2 &= \sinh\frac{\tau}{2}\cosh\frac{\chi-\phi}{2} \\
x_3 &= \sinh\frac{\tau}{2}\sinh\frac{\chi-\phi}{2} \,.
\end{aligned} \tag{A.4}$$

We see that $x_0^2 - x_1^2 = \cosh^2\frac{\tau}{2}$ and $x_3^2 - x_2^2 = -\sinh^2\frac{\tau}{2}$. The last Euler parametrization which we use is

$$g = e^{i\chi \frac{\sigma_2}{2}} e^{\rho \frac{\sigma_3}{2}} e^{i\phi \frac{\sigma_2}{2}} \,, \tag{A.5}$$

which is connected to (4.8) by formulae

$$\begin{aligned} x_0 &= \cosh\frac{\rho}{2}\cos\frac{\chi+\phi}{2} \\ x_1 &= \sinh\frac{\rho}{2}\cos\frac{\chi-\phi}{2} \\ x_2 &= -\sinh\frac{\rho}{2}\sin\frac{\chi-\phi}{2} \\ x_3 &= \cosh\frac{\rho}{2}\sin\frac{\chi+\phi}{2}\,. \end{aligned} \tag{A.6}$$

We see that $x_0^2+x_3^2=\cosh^2\frac{\rho}{2}$ and $x_1^2+x_2^2=\sinh^2\frac{\rho}{2}$. These coordinates also are known as the cylindrical coordinates.

References

[1] M.R. Gaberdiel, A. Recknagel and G.M.T. Watts, *The conformal boundary states for SU(2) at level 1*, *Nucl. Phys.* **B 626** (2002) 344 [hep-th/0108102].

[2] M.R. Gaberdiel and A. Recknagel, *Conformal boundary states for free bosons and fermions*, *J. High Energy Phys.* **11** (2001) 016 [hep-th/0108238].

[3] R.A. Janik, *Exceptional boundary states at c = 1*, *Nucl. Phys.* **B 618** (2001) 675 [hep-th/0109021].

[4] A. Cappelli and G. D'Appollonio, *Boundary states of c = 1 and 3/2 rational conformal field theories*, *J. High Energy Phys.* **02** (2002) 039 [hep-th/0201173].

[5] J.M. Maldacena, G.W. Moore and N. Seiberg, *Geometrical interpretation of D-branes in gauged WZW models*, *J. High Energy Phys.* **07** (2001) 046 [hep-th/0105038].

[6] J.M. Maldacena, G.W. Moore and N. Seiberg, *D-brane instantons and K-theory charges*, *J. High Energy Phys.* **11** (2001) 062 [hep-th/0108100].

[7] T. Quella and V. Schomerus, *Symmetry breaking boundary states and defect lines*, *J. High Energy Phys.* **06** (2002) 028 [hep-th/0203161].

[8] A.Y. Alekseev and V. Schomerus, *D-branes in the WZW model*, *Phys. Rev.* **D 60** (1999) 061901 [hep-th/9812193].

[9] A. Rajaraman, *New AdS_3 branes and boundary states*, hep-th/0109200.

[10] C. Klimčík and P. Ševera, *Open strings and D-branes in WZNW models*, *Nucl. Phys.* **B 488** (1997) 653 [hep-th/9609112].

[11] K. Gawędzki, *Conformal field theory: a case study*, in *Conformal field theory, frontier in physics* 102, Y. Nutku, C. Saclioglu, T. Turgut eds., Perseus Publishing, 2000 [hep-th/9904145].

[12] S. Elitzur and G. Sarkissian, *D-branes on a gauged WZW model*, *Nucl. Phys.* **B 625** (2002) 166 [hep-th/0108142].

[13] E. Kiritsis, *Exact duality symmetries in CFT and string theory*, *Nucl. Phys.* **B 405** (1993) 109 [hep-th/9302033].

[14] S. Stanciu, *A note on D-branes in group manifolds: flux quantization and d0-charge*, *J. High Energy Phys.* **10** (2000) 015 [hep-th/0006145].

[15] P. Bordalo, S. Ribault and C. Schweigert, *Flux stabilization in compact groups*, *J. High Energy Phys.* **10** (2001) 036 [hep-th/0108201].

[16] C. Bachas, M.R. Douglas and C. Schweigert, *Flux stabilization of D-branes*, *J. High Energy Phys.* **05** (2000) 048 [hep-th/0003037].

[17] S. Elitzur, A. Giveon, D. Kutasov, E. Rabinovici and G. Sarkissian, *D-branes in the background of NS fivebranes*, *J. High Energy Phys.* **08** (2000) 046 [hep-th/0005052].

[18] N.J. Vilenkin, *Special functions and the theory of group representations*, AMS.